General equilibrium with substitutes and complements

Consider two individuals. Individual A has the following utility function:

$$u_A = min\{x_A, y_A\}$$

Individual B has the following utility function:

$$u_B = 2x_B + y_B$$

And the endowments are $\omega_A = (10, 6)$, $\omega_B = (6, 4)$. Find the equilibrium demands and prices.

Solution

To find the equilibrium demands and prices, it is required that both individuals are maximizing utility and that at the same time one of the two market clearing conditions is met, that is, the sum of demands equals the available amount of endowments of a good. First, we obtain the Marshallian demand functions of individual A:

$$x_A = y_A$$

We insert into the budget constraint and rearrange:

$$x_A p_x + y_A p_y = m$$

$$y_A p_x + y_A p_y = m$$

$$y_A(p_x + p_y) = m$$

$$y_A = \frac{m^A}{p_x + p_y}$$

And since $x_A = y_A$

$$x_A = \frac{m^A}{p_x + p_y}$$

On the other hand, for individual B, they have split Marshallian demand functions:

$$x_B = \begin{cases} m^B/p_x & \text{if } 2 > \frac{p_x}{p_y} \\ 0 & \text{if } 2 < \frac{p_x}{p_y} \\ [0, m^B/p_x] & \text{if } 2 = \frac{p_x}{p_y} \end{cases}$$

$$y_B = \begin{cases} m^B/p_y & \text{if } 2 < \frac{p_x}{p_y} \\ 0 & \text{if } 2 > \frac{p_x}{p_y} \\ [0, m^B/p_y] & \text{if } 2 = \frac{p_x}{p_y} \end{cases}$$

Normalizing $p_y = 1$ and replacing m^A and m^B with their definitions:

$$y_A = \frac{10p_x + 6}{p_x + 1}$$

$$x_A = \frac{10p_x + 6}{p_x + 1}$$

$$x_B = \begin{cases} \frac{5p_x+4}{p_x} & \text{if } 2 > \frac{p_x}{p_y} \\ 0 & \text{if } 2 < \frac{p_x}{p_y} \\ [0, \frac{5p_x+4}{p_x}] & \text{if } 2 = \frac{p_x}{p_y} \end{cases}$$

$$y_B = \begin{cases} \frac{5p_x+4}{1} & \text{if } 2 < \frac{p_x}{p_y} \\ 0 & \text{if } 2 > \frac{p_x}{p_y} \\ [0, \frac{5p_x+4}{1}] & \text{if } 2 = \frac{p_x}{p_y} \end{cases}$$

Assuming $2 > p_x/p_y$ and seeing if we are in the first branch, we use the market clearing conditions:

$$y_A + y_B = 6 + 4$$

$$\frac{10p_x + 6}{p_x + 1} + 0 = 10$$

$$10p_x + 6 = 10p_x + 10$$

We obtain a contradiction, so we go with another case: $2 < p_x/p_y$

$$x_A + x_B = 10 + 6$$

$$\frac{10p_x + 6}{p_x + 1} + 0 = 16$$

$$10p_x + 6 = 16p_x + 16$$

$$p_x = -10/6$$

Again a contradiction, we go for the last case: $2 = p_x/p_y$ which implies that $p_x = 2$

$$\frac{10 * 2 + 6}{2 + 1} + x_B = 16$$

$$x_B = 16 - 26/3$$

$$x_B = 22/3$$

On the other hand:

$$y_B = 6 * 2 + 4 * 1 - (22/3) * 2$$

$$y_B = 4/3$$

$$x_A = y_A = 26/3$$

We check that the market clearing conditions are met:

$$26/3 + 22/3 = 16$$

$$26/3 + 4/3 = 10$$